Assignment 5

**PORTFOLIO SELECTION AND THE CAPITAL ASSET PRICING MODEL**

# Step 1:

**Portfolio Selection**

For the initial analysis, I focused on the realized returns and risk for each stock, the S&P 500 index, and an equally weighted portfolio of the three stocks. I calculated the mean return and standard deviation of each asset's realized returns. This provides a comparison of each asset's average performance and the associated volatility, or risk.

**Mean Comparisons**:

* Maya has the highest monthly return at 1.08%, indicating strong performance, followed by Urban and the portfolio, both at 0.83%. Harris shows the lowest return at 0.58%.
* The portfolio’s mean return aligns with Urban's, providing a middle-ground result between the highest- and lowest-return stocks.

**Standard Deviation Comparisons**:

* Maya’s standard deviation is the highest at 12.57%, showing significant volatility. Harris has the lowest volatility at 5.79%, making it the most stable.
* The portfolio’s standard deviation of 6.21% demonstrates diversification benefits, offering moderate risk—lower than each of the individual, high-risk stocks.

**Investment Insights:**

* **For a Single Stock Investment:** Maya’s higher return pairs with high risk, potentially suitable for a more aggressive investor. Harris, with its lower risk and stability, would suit a conservative profile.
* **Portfolio Benefit:** The equally weighted portfolio balances return and risk at 0.83% and 6.21%, respectively, making it a suitable choice for both investor types, depending on tolerance for moderate risk.

**Considering the S&P 500 Index:**

* Adding the S&P 500 as an investment option provides an attractive alternative with a moderate mean return of 0.75% and lower risk (4.33%) than the portfolio and individual stocks.

# Step 2:

I constructed the efficient frontier for a portfolio comprising two assets: Urban and Maya. Using incremental weights from 0% to 100% for each asset, I calculated the expected return and standard deviation for each portfolio combination. The goal was to identify how various weight allocations impact the risk-return trade-off.

**Calculations:**

1. **Portfolio Return**: Formula: rp=w1⋅r1+w2⋅r2r
2. **Portfolio Standard Deviation**: Formula:

σp =SQRT(w12⋅σ12+w22⋅σ22+2⋅w1⋅w2⋅σ1⋅σ2⋅corr(1,2))

The Portfolio Risk vs. Return graph shows a curved line representing the efficient frontier for combinations of Urban and Maya. The x-axis denotes the portfolio’s standard deviation (risk), while the y-axis represents the expected returns. The plot demonstrates the risk-return trade-off, with portfolios on the left exhibiting lower risk and portfolios on the right displaying higher expected returns.

The curve generated by plotting portfolio returns against portfolio standard deviations for the stock pairs has a concave, “bowed” shape. This shape illustrates the benefits of diversification: as we adjust weights between the two stocks, the portfolio achieves lower risk for a given level of return compared to holding either stock alone. The curve represents the set of efficient portfolios, showing how risk decreases as we move from a single-stock portfolio to a balanced mix. The concavity indicates that combining two stocks with less-than-perfect correlation improves the risk-return trade-off.

**Effect of Correlation on the Curve’s Shape**: Correlation significantly affects the curve’s shape. When correlation is high (close to 1), the curve becomes almost linear, indicating limited diversification benefits because the stocks tend to move in tandem. In this case, the risk reduction from combining the stocks is minimal, and the portfolio doesn’t achieve much lower volatility than the individual stocks. When correlation is lower (approaching 0 or negative), the curve becomes more pronounced (more “bowed”), demonstrating greater diversification benefits. With a low or negative correlation, combining the stocks offsets some of each other’s risks, reducing overall volatility more effectively. A correlation of -1 would theoretically allow for perfect diversification, potentially eliminating all risk at certain weight combinations.

**Interpretation of Stock Correlations**:

The correlation between individual stocks indicates how they move relative to each other. In a portfolio context, lower correlation between stocks improves the risk-return profile, while higher correlation suggests the stocks provide limited diversification benefits.

Correlation between the Equally Weighted Portfolio and the S&P 500 shows how closely the portfolio’s performance aligns with the broader market. A high correlation suggests that the portfolio mimics market movements and may not offer unique risk reduction relative to the market. A lower correlation implies that the portfolio has independent performance characteristics, providing a distinct diversification advantage, especially valuable for investors seeking less exposure to market-wide risks.

# Step 3:

I expanded the portfolio to include three stocks: Harris, Urban, and Maya. I analysed all possible weights combinations of these stocks and plotted the expected return against the portfolio’s standard deviation, resulting in a curved shape that demonstrates the relationship between risk and return for a three-asset portfolio.

I plotted expected returns against standard deviations for each possible combination, resulting in a concave, upward-sloping curve. This curve, known as the efficient frontier, represents portfolios that offer the highest expected return for each level of risk.

Portfolios positioned along the efficient frontier are optimal because they maximize return for a given risk level. Moving along the curve from left to right, risk increases, but so does the potential return, making it suitable for various investor profiles.

As the weight shifts towards Harris (the lowest-risk stock), the portfolio standard deviation decreases, showing lower volatility. Meanwhile, increasing the allocation towards Maya raises both expected returns and risk due to Maya’s higher individual return and volatility.

**Portfolio Recommendations**:

* **For the 28-year-old Manager**: I recommend a portfolio closer to the high-return, high-risk end of the efficient frontier. This might involve a higher allocation to Maya, allowing for potentially higher gains in exchange for greater volatility, which aligns with the manager’s likely higher risk tolerance and investment horizon. Example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Weight of Harris | Weight of Urban | Weight of Maya | Portfolio Return | Portfolio Standard Deviation |
| 0.14 | 0.07 | 0.79 | 0.991607013 | 10.18993423 |

* **For the 60-year-old Retiree**: A portfolio closer to the low-risk end of the efficient frontier would be preferable, likely weighted more heavily towards Harris. This allocation minimizes volatility while still generating returns, which is suitable for a retiree focused on capital preservation. Example :

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Weight of Harris | Weight of Urban | Weight of Maya | Portfolio Return | Portfolio Standard Deviation |
| 0.51 | 0.29 | 0.2 | 0.75235876 | 5.379242849 |

**S&P 500 Index Position**:

* The S&P 500 index, with a return of 0.75% and standard deviation of 4.33%, generally lies below or to the left of the efficient frontier. This indicates that the index is a more stable, diversified option but may not offer as high a return as some stock combinations.
* For the retiree, the S&P could be an appealing alternative, providing steady returns with lower risk. For the young manager, combining the S&P with higher-return stocks could further enhance return potential while still maintaining a balance with market stability.

# Step 4:

Adding a risk-free bond to our investment options enhances the diversification benefits. The bond’s low, stable return of 0.45% per month provides an anchor point of zero risk in our portfolio, allowing us to construct a range of risk-return profiles by combining it with the market portfolio of stocks.

By combining this risk-free asset with the market portfolio, we generate the Capital Market Line (CML), which represents the highest possible returns at each level of risk when mixing risky assets with a risk-free asset. The CML is a straight line, sloping upwards from the risk-free rate to the point of tangency with the efficient frontier of stock portfolios. This line is straight because each increase in risk (standard deviation) correlates linearly with higher returns when combining the risk-free asset with the optimal risky portfolio.

**Recommendations for Investors:**

1. **Risk-Averse Investor**: They should allocate a higher portion to the risk-free bond and a smaller portion to the market portfolio, staying on the lower end of the CML for minimal risk while earning returns above what any single stock combination would yield.
2. **Risk-Seeking Investor**: They should leverage the CML by borrowing at the risk-free rate (if possible) and investing more in the market portfolio, positioning on the upper end of the CML for higher returns with higher risk.

For our two hypothetical investors, the 28-year-old new manager and the 60-year-old retiree, consider a specific strategy based on the risk profiles and investment horizons typical for their life stages.

1. **28-Year-Old Newly Promoted Manager**
   * **Profile**: Young, long investment horizon, able to recover from losses, and seeking higher returns.
   * **Recommendation**: Position on the upper end of the Capital Market Line (CML), combining a large share of the market portfolio with possible leveraged positions.
   * **Allocation:** 80–90% in the market portfolio and 10–20% in a risk-free bond. If leveraging, this could exceed 100% in the market portfolio.
   * **Rationale:** With time to absorb volatility, this investor can prioritize growth. This CML approach offers higher returns than a stock-only portfolio, with manageable risk.
2. **60-Year-Old Retiree**
   * **Profile:** Retired, risk-averse, prioritizing capital preservation and stable income.
   * **Recommendation:** Position on the lower end of the CML, focusing on stability with a mix of bonds and the market portfolio.
   * **Allocation:** 60–70% in a risk-free bond and 30–40% in the market portfolio to ensure growth potential with minimal risk.
   * **Rationale:** A high bond allocation reduces volatility, aligning with a preservation strategy that ensures steady returns without significant market exposure.

# Step 5:

The regression results provide with key insights into the risk and return characteristics of each stock relative to the S&P 500, which serves as the market benchmark. Below is an analysis of each stock's beta and alpha values, along with recommendations for each investor.

**Alpha and Beta Analysis**

1. **Harris**
   * **Alpha**: 0.2872
   * **Beta**: 0.3919
2. **Urban**
   * **Alpha**: 0.1149
   * **Beta**: 0.9416
3. **Maya**
   * **Alpha**: 0.0946
   * **Beta**: 1.3043

**Risk Analysis Based on Beta**

* **Most Risky:** Maya. Highest beta (1.3043). It is more volatile than the market, indicating greater sensitivity to market movements.
* **Least Risky:** Harris. Beta of 0.3919. It shows the least market sensitivity, offering a more stable, low-risk profile.

**Comparison to Standard Deviations**

The beta analysis aligns with our previous findings based on standard deviations, identifying Maya as the most volatile and Harris as the least. While standard deviation captures total risk, beta isolates market-related risk, clarifying each stock's sensitivity to market changes.

**Recommendations for Hypothetical Investors**

1. **28-Year-Old Investor (Higher Risk Tolerance)**
   * **Recommendation**: Allocate a larger portion to Maya for market-driven growth potential, given Maya’s higher beta.
   * **Diversification**: Add some Urban exposure to reduce volatility while maintaining market alignment.
2. **60-Year-Old Retired Investor (Lower Risk Tolerance)**
   * **Recommendation**: Harris is the most suitable, offering stability with its low beta.
   * **Diversification**: A small Urban allocation could add moderate growth without excessive risk.

**Economic Meaning of Alpha**

Alpha represents the stock’s returns independent of market movements. Positive Alpha across all stocks suggests that each performs better than expected given its beta, indicating intrinsic value or firm-specific strengths. This is favourable for conservative investors seeking stable, intrinsic growth, though younger investors may focus more on beta for market-driven returns.

# Step 6:

The realized returns for each stock exceed their CAPM-predicted expected returns, suggesting that each stock has outperformed relative to its risk-adjusted expectation.

**Comparison of Realized vs. Expected Returns**

* **Harris**: Expected return of 5.78% (annual) vs. realized return of 7.22%
* **Urban**: Expected return of 6.13% vs. realized return of 10.37%
* **Maya**: Expected return of 6.36% vs. realized return of 13.74%

Each stock’s realized return is significantly higher than what CAPM predicts based on its beta, indicating either favourable firm-specific factors or market conditions that boosted returns beyond market-driven expectations.

**Market Response if Expected Returns Deviate from CAPM Predictions**

Assuming CAPM accurately represents the appropriate expected returns for these stocks, a sustained deviation between realized and expected returns would typically lead to price adjustments:

* **If Realized Returns Exceed Expected Returns**: Higher-than-expected realized returns suggest that the stocks are delivering more return for their level of risk than CAPM predicts. If investors expect this outperformance to continue, demand for the stocks will increase, pushing prices up. As prices rise, future expected returns will adjust downward, moving closer to the CAPM-predicted level.
* **If Realized Returns Fall Below Expected Returns**: Conversely, if realized returns are persistently lower than CAPM expectations, it suggests that the stocks are underperforming given their risk levels. Investors may respond by selling these stocks, reducing their prices. This price decline would lead to higher expected future returns, gradually bringing them back in line with CAPM predictions.